NOTATION

j, mass filtration velocity; p, fluid pressure, Pa; T, temperature; i, enthalpy; ρ, fluid density, kg/m^3; μ, fluid viscosity, N · sec/m^2; m, porosity of the medium; k, permeability of the porous medium, m^2; λ, thermal conductivity of the fluid–porous-medium system, W/m · deg; C, volumetric specific heat of the porous medium, J/(m^3 · deg); C_p, specific heat of the gas at constant pressure, J/(kg · deg); C', specific heat of the liquid, J/(kg · deg); K, compressive bulk modulus, N/m^2; α, coefficient of cubical expansion, deg^{-1}.

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STEADY-STATE PROBLEM OF LOCAL PORE COOLING

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The temperature field in a porous half-space with filtration of coolant from a source is examined.

Pore cooling has come into use in recent years in several sectors of modern industry to protect various structural elements from high heat fluxes. The high efficiency of this method of cooling is due to the developed surface with which the coolant is in contact during its motion through the porous medium. As a result of this, heat is absorbed, and the boundary layer at the leakage surface is transformed in such a way that heat transfer from the high-temperature gas flow to the wall being protected is reduced.

Together with the continuous supply of coolant through the wall [1], in our opinion coolant can also be supplied to certain local zones in some cases. This produces zonal pore cooling and creates the thermal regime required for the most heavily thermally stressed sections.

An important task in designing such systems is studying the temperature fields inside porous materials with allowance for the filtration processes occurring. To solve this problem, it is first necessary to construct the solution of the two-dimensional filtration problem and obtain the pressure distribution in the porous body. The heat-transfer equation can then be used with this data to find the temperature field.

Let us examine this problem using the example of coolant flow in a porous half-space (Fig. 1a [2]). We will assume that the cooling gas is moving in an undeformed, uniformly porous medium from a source of intensity 2M, located at point A, to the leakage surface. Constant pressure p_2 and temperature T_2 are maintained at the leakage surface, while the pressure and temperature at the source are p_1 and T_1, respectively. The thermophysical characteristics of the gas and the porous material are assumed to be constant, and equality is maintained between the temperatures of the body and the coolant at any point of the filtration region.

The gas flow in the porous medium obeys the resistance law

$$\frac{\alpha}{\mu(T)} \operatorname{grad} p = -\frac{f(V)}{V} \bar{V}. \quad (1)$$

The process of heat and mass transfer is described by the equations:

$$\lambda \Delta T - c\rho \bar{V} \operatorname{grad} T = 0, \quad (2)$$

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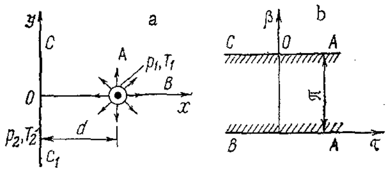

Fig. 1. Diagram of the region of coolant flow in the porous half-space.

$$\frac{\partial\psi^*}{\partial\beta}=\frac{\sqrt{n+1}}{\chi}\exp(-2\varepsilon\tau)\frac{\partial P^*}{\partial\tau},\quad \frac{\partial\psi^*}{\partial\tau}=-\frac{\sqrt{n+1}}{\chi}\exp(-2\varepsilon\tau)\frac{\partial P^*}{\partial\beta}. \tag{3}$$

We employed a hodograph transformation of the filtration rate, transforming the investigated region in the plane τ, β (Fig. 1b) into an infinite strip where there exists a certain function $Q(\tau, \beta)$ which satisfies the Helmholtz equation. Successive use of Fourier's theory of generalized integrals, the theory of analytic functions, and the Weiner–Hopf method to solve this equation allows us to obtain the following relations for the sought function [2]:

at $\tau > 0$

$$Q(\tau,\beta)=\frac{\beta}{\pi}\exp(\varepsilon\tau)+\Phi(\varepsilon)\sum_{k=1}^{\infty}(-1)^k\frac{k\Phi(r_k)\exp(-r_k\tau)}{r_k(r_k+\varepsilon)}\sin k\beta, \tag{4}$$

at $\tau < 0$

$$Q(\tau,\beta)=\frac{\Phi(\varepsilon)}{\pi}\sum_{k=1}^{\infty}(-1)^k\frac{\exp(S_k\tau)}{S_k(S_k-\varepsilon)\cdot\Phi(S_k)}\sin\frac{2k-1}{2}\beta, \tag{5}$$

where

$$\Phi(\varepsilon)=\frac{1}{\pi}\prod_{k=1}^{\infty}\frac{2k}{2k-1}\,\frac{\varepsilon-S_k}{\varepsilon-r_k};\quad r_k=\sqrt{k^2+\varepsilon^2};$$

$$S_k=\sqrt{\frac{(2k-1)^2}{4}+\varepsilon^2};\quad \Phi(Z)=\prod_{k=1}^{\infty}\frac{2k-1}{2k}\,\frac{Z+r_k}{Z+S_k}.$$

The relation

$$dx^*=-\left[\frac{1}{\chi}\exp(-\sqrt{n+1}\,\tau)\cos\beta dP^*+\exp\left(-\frac{\tau}{\sqrt{n+1}}\right)\sin\beta d\psi^*\right] \tag{6}$$

permits us to change over from τ to the physical coordinate x^*.

Allowing for (4), (5), we find the pressure distribution from the source to the leakage surface with $\beta = \pi$, here integrating the relation

$$dP^*=\frac{\partial P^*}{\partial\tau}d\tau+\frac{\partial P^*}{\partial\beta}d\beta$$

with the use of system (3) and the substitution

$$\psi^*=Q\exp(-\varepsilon\tau). \tag{7}$$

As a result, we have

$$\frac{P^*}{\chi}=\frac{1}{\pi\sqrt{n+1}}\left\{\frac{1}{2\varepsilon}[\exp(2\varepsilon\tau)-1]-\pi\Phi(\varepsilon)\sum_{k=1}^{\infty}\frac{k^2\Phi(r_k)\exp[-\tau(r_k-\varepsilon)]-1}{r_k(r_k^2-\varepsilon^2)}\right\}. \tag{8}$$

Equation (8) can be represented in the plane of the physical coordinates by means of the relation $x^* = f(\tau)$ with $\beta = \pi$. Integrating (6) from 0 to ∞ with the use of (3), (4), and (7), we find

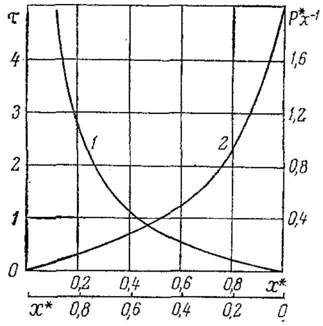

Fig. 2

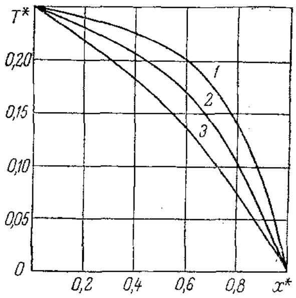

Fig. 3

Fig. 2. The relations with n = 1: 1) $P^*\chi^{-1} = f(x^*)$; 2) $x^* = f(\tau)$.

Fig. 3. Temperature change along the x* axis ($M = 6 \cdot 10^{-4}$ m²/sec, λ = 3 W/m · deg) with the flow of: 1) air ($K_n = 0.15$); 2) helium ($K_n = 0.1$); 3) hydrogen ($K_n = 0.15$).

$$x^* = -\frac{1}{\pi\sqrt{n+1}}\left\{\sqrt{n+1}\left[\exp\left(-\frac{\tau}{\sqrt{n+1}}\right)-1\right]+\pi\Phi(\varepsilon)\sum_{k=1}^{\infty}\frac{k^2\Phi(r_k)\exp[-\tau(r_k+\eta)]-1}{r_k(r_k+\varepsilon)(r_k+\eta)}\right\}, \tag{9}$$

where $\eta = (n+2)/2(n+1)^{1/2}$. The graphical description of (8), (9) with n = 1 by means of a computer is shown in Fig. 2.

We obtain the temperature field distribution inside the porous half-space in the flow of an ideal gas from the solution of Eq. (2). This solution, with allowance for (1), can be represented as follows in curvilinear orthogonal coordinates p, ψ (pressure, stream function) [3]:

$$\frac{\partial}{\partial p}\left[\frac{R^{n+1}\mu(T)\,T^{n+1}f(V)}{\alpha p^{n+1}V}\,\frac{\partial T}{\partial p}\right]+\frac{\partial}{\partial\psi}\left[\frac{\alpha p^{n+1}V}{R^{n+1}\mu(T)\,T^{n+1}f(V)}\,\frac{\partial T}{\partial\psi}\right]+\frac{c}{\lambda}\,\frac{\partial T}{\partial p}=0. \tag{10}$$

As shown in [3], for a linear resistance law the temperature is a single-valued function of the pressure, and (10) is transformed into an ordinary differential equation which can be easily integrated. With nonlinear filtration, the problem is complicated considerably. However, if we approximately assume $\xi = (fV)/V \simeq$ const, then, as before, we can suppose that the temperature is a single-valued function of the pressure and, instead of (10), write

$$\frac{d}{dp}\left[\frac{R^{n+1}\mu(T)\,T^{n+1}\xi}{\alpha p^{n+1}}\,\frac{dT}{dp}\right]+\frac{c}{\lambda}\,\frac{dT}{dp}=0. \tag{11}$$

Integrating (11), we obtain

$$dp=\frac{R^{n+1}\,T^{n+1}\,\mu(T)}{Fp^{n+1}(D-T)}\,dT, \tag{12}$$

where $F = c\alpha/\lambda\xi$; D is the constant of integration.

It is physically evident that the presence of a single point source in the investigated region presumes a value $p_1 = \infty$. Then, assuming that $p_2 = 0$ on the leakage surface and then integrating (12), we find that $D = T_1$. Having made the substitution

$$dP=\frac{p^{n+1}}{T^{n+1}\,R^{n+1}\,\mu(T)}\,dp,$$

instead of (12) we obtain

$$dp=\frac{dT}{F(T_1-T)}.$$

From which we have

$$T = T_1 - (T_1 - T_2) \exp(-FP). \tag{13}$$

Equation (13), with allowance for the pore cooling criterion [4] and the dimensionless temperature $T^* = (T - T_1)/T_1$, is reduced to the form

$$T^* = T_2^* \exp\left(-K_n \frac{P^*}{\chi} \frac{V_0}{\rho\xi}\right).$$

Figure 3 shows the temperature distribution in the porous half-space in the flow of air, helium, and hydrogen. It can be seen from the graph that, to maintain a certain thermal state in the porous body during cooling with a gas, it is best to use a substance with a lower molecular weight.

NOTATION

T, temperature; c, specific heat of the gas; ρ, density of the gas; λ, effective thermal conductivity of the porous-body–coolant system; τ, β, Chaplygin variables; n + 1, degree of filtration (filtration is linear at n = 0); $\chi = V_0^n M/P_0\alpha$, dimensionless filtration parameter; $\psi^* = \psi/M$, dimensionless stream function; $P^* = P/P_0$, dimensionless pressure; α, constant characterizing the porous medium and coolant; $\mu(T)$, absolute viscosity of the gas; f(v), function determining the filtration law in each specific case; $x^* = x/d$, dimensionless coordinate; d, characteristic dimension; R, gas constant.

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THEORY OF THE FLOW AND CONDUCTION OF INHOMOGENEOUS MEDIA

I. BASIC MODEL OF AN INHOMOGENEOUS MEDIUM

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A basic model of an inhomogeneous medium is outlined and, by a combination of the methods of flow theory and reduction to an elementary cell, an analytic dependence is obtained for the conduction of such a medium.

In studying the conduction of inhomogeneous materials with a random distribution of components, there has been steadily increasing use, in recent years, of a new method of investigation, called flow theory [1-3]. For a binary inhomogeneous system, in which the conductivity of one component $\Lambda_1 \neq 0$ is nonzero, while the other is zero $\Lambda_2 = 0$, the effective conductivity Λ, according to flow theory, is [3]

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